Linearization of Volterra series based on first order Taylor series expansion

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***Abstract*—One of the most commonly used mathematical models for representing nonlinear systems with memory is the Volterra series. The main objective of this study is to model nonlinear electric circuits using the Volterra series with two inputs. In this work, we will compare two approaches for obtaining the Volterra series using one input as a small signal: one applies the small signal theory directly in a two input Volterra series, and the other uses a Taylor series expanded around the large signal input and truncated at first order. For result analysis, the Mean Square Error (MSE) will be measured, along with evaluating the complexity of each model based on the number of coefficients employed. The results obtained demonstrate that the first model exhibits a remarkable level of accuracy, as indicated by an MSE of $8.0823 \cdot 10^{-07}$. In contrast, the second model yields an MSE of $3.5188 \cdot 10^{-05}$. However, the second model has the advantage that it does not require the output of the Volterra series with two inputs to determine the coefficients.**

***Index Terms*—mathematical modeling, Volterra, linear regression.**

I. Introduction

Mathematical modeling serves as a valuable tool across various fields of knowledge [1], allowing the description of events through mathematical and physical resources. One such model, the Volterra series, stands out as a nonlinear mathematical model with memory that finds application in modeling nonlinear electric circuits, such as power amplifiers (PAs). The versatility of this model extends to simulating and analyzing the performance of radio frequency power amplifiers under two-tone signal conditions, where the second tone exhibits reduced amplitude. Moreover, it proves instrumental in simulating the behavior of equipment within power electrical systems characterized by a dominant high-amplitude fundamental frequency and accompanying lower-amplitude harmonic components.

In this study, we focus on investigating the Volterra series with two inputs and one output, examining two distinct methods of obtaining it when one input represents a small signal. The first method involves employing the small signal theory directly in a two input Volterra series, which yields satisfactory accuracy according to [2]. The second method, the primary focus of this research, involves the expansion of a Volterra series with one input by a first order Taylor series.

The analysis will be done on top of the number of coefficients provided by each model, thus measuring the complexity and the mean square error (MSE) will also be evaluated to verify the accuracy of the measurements.

II. Mathematical Modeling Using Volterra Series

Mathematical modeling aims to describe various phenomena using mathematical techniques [1]. One such model is the Volterra series, which describes nonlinear systems with memory [3]. A model with memory implies that the current output is influenced by previous states [4].

The Volterra series with two inputs and one output offers greater accuracy compared to the Volterra series with one input and one output. However, the former is more complex [5]. In this study, we analyze two methods for obtaining the Volterra series with two inputs, with the objective of reducing complexity.

The first approach involves applying the small signal theory when a small signal is present in one of the inputs. This method has proven to be effective and yields satisfactory results [2]. The second approach is to use a truncated Taylor series of first order. This model does not require the output of the Volterra series with two inputs to obtain the coefficients. Instead, we can estimate the output of the Volterra series with two inputs using: the input and output of the Volterra series with one input and one output and one input of the Volterra series with two inputs and one output. The relationship between the inputs and outputs of the models can be observed through the block diagram depicted in Figure 1.

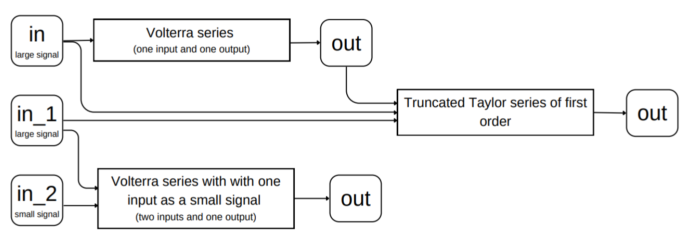

Fig. 1. Block diagrams of Volterra series used in this work.

Although the Volterra series is a nonlinear model, its coefficients are related in a linear way. This property allows us to extract these coefficients efficiently using the Least Square

Method (LSM)[6]. The coefficients obtained through LSM are then used in the Volterra series equation. To assess the accuracy of the model, the MSE is employed. The MSE quantifies the average squared difference between the predicted values and the corresponding actual values. A smaller MSE indicates a result closer to the expected values, reflecting a higher level of accuracy. Comparing the MSE values obtained from different models enables us to evaluate their performance and identify the model that provides the best fit to the data.

III. Development Of Models Based On Volterra Series

The equation for the Volterra series with one input and one output is given by:

$$y(n) = f[x(n), \ldots, x(n-M))] =$$

$$= \sum_{p=1}^{P_0} \sum_{q_1=0}^{M} \cdots \sum_{q_p=q_{p-1}}^{M} C_{p(q_1,q_2,\ldots,q_p)} \prod_{j=1}^{p} x(n-q_j) \qquad (1)$$

in (1), $y(n)$ represents the output at time instant n, P_0 denotes the polynomial order, M represents the memory size, $C_{p(q_1,q_2,\ldots,q_p)}$ corresponds to the coefficients, and $x(n)$ represents the input. In this work the polynomial order was fixed in 3 and the largest memory is set to 1, what requires us to extract 9 coefficients. This equation was used to formulate the equation for the Volterra series with one input and one output expanded as a truncated Taylor series of first order that is given by:

$$y(n) = f[x(n), x(n-1)]\Big|_{x(n)=x_0(n), x(n-1)=x_0(n-1)} + \frac{\partial f[x(n), x(n-1)]}{\partial x(n)}\Big|_{x(n)=x_0(n), x(n-1)=x_0(n-1)} \cdot [x(n) - x_0(n)] + \frac{\partial f[x(n), x(n-1)]}{\partial x(n-1)}\Big|_{x(n)=x_0(n), x(n-1)=x_0(n-1)} \cdot [x(n-1) - x_0(n-1)] \qquad (2)$$

where $f[x(n), x(n-1)]$ corresponds to (1), and $x_0(n)$ and $x_0(n-1)$ represent the large input in a system that contains two inputs. Having it in mind, on (2) the difference $[x(n) - x_0(n)]$ and the difference $[x(n-1) - x_0(n-1)]$ represent the second input of a Volterra series of two inputs. So $y(n)$ model a non-linear system with two inputs, giving us the output of a Volterra series of two inputs and one output. The model coeficients of (2) are exactled the same as the model coefficients of (1). The other approach, discussed in [2], starts form a general two input and one output Volterra series, given by:

$$y(n) = \sum_{p_1=0}^{P_0} \sum_{p_2=0}^{P_0} \sum_{m_1,1=0}^{M} \cdots \sum_{m_1,p_1=m_1,p_1-1}^{M} \sum_{m_2,1=0}^{M} \cdots \sum_{m_2,p_2=m_2,p_2-1}^{M} C_{p_1,p_1,(m_1,p_1),\ldots,(m_2,p_2)} \prod_{q_1=1}^{p_1} x_1(n-m_{1,q_1}) \prod_{q_2=1}^{p_2} x_2(n-m_{2,q_2}) \qquad (3)$$

In (3), if P_0 is fixed to 3 and M is equal to 1, there are 33 coefficients. Imposing the small signal theory in the input x_2, the resulting Volterra series in given by:

$$y(n) = \sum_{p_1=0}^{P_0} \sum_{p_2=0}^{1} \sum_{m_1,1=0}^{M} \cdots \sum_{m_1,p_1=m_1,p_1-1}^{M} \sum_{m_2,1=0}^{M} \cdots \sum_{m_2,p_2=m_2,p_2-1}^{M} C_{p_1,p_1,(m_1,p_1),\ldots,(m_2,p_2)} \prod_{q_1=1}^{p_1} x_1(n-m_{1,q_1}) \prod_{q_2=1}^{p_2} x_2(n-m_{2,q_2}) \qquad (4)$$

In (4), if P_0 is fixed to 3 and M is equal to 1, there are 20 coefficients.

IV. Results and Discussion

To implement the models of (2) and (4), plot wave form, manipulate algebraic systems and do all the calculations the MATLAB was used. The input and output data to be applied to the model given by (1) was obtained using the circuit shown in Figure 2. In this circuit, the output was measured across a 10 Ω resistor, while the input was provided by the source V_{in} with frequency of 1 kHz.

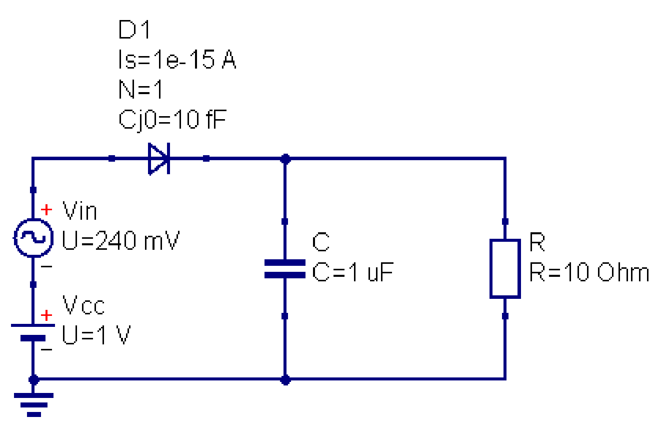

Fig. 2. Circuit configuration used to obtain the data for (1).

To extract the model of (4), the circuit depicted in Figure 3 was employed. In this circuit, the inputs were sourced from V_{in1} and V_{in2} with the same frequency of 1 kHz, respectively.

Similarly, the output was measured across the same 10 Ω resistor.

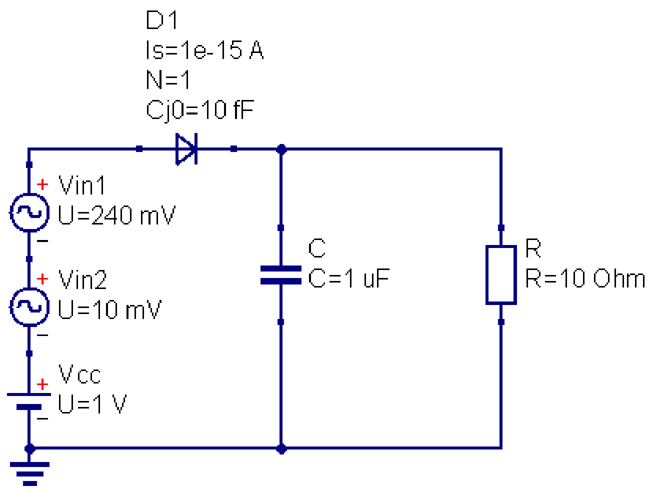

Fig. 3. Circuit configuration used to obtain the data for (4).

By employing these circuit configurations, the desired data was collected and used to implement the two models. The Figure 4 represents (1) and the Figure 5 represents (2), both implemented with obtained database. These figures compare the estimated output with the desired output.

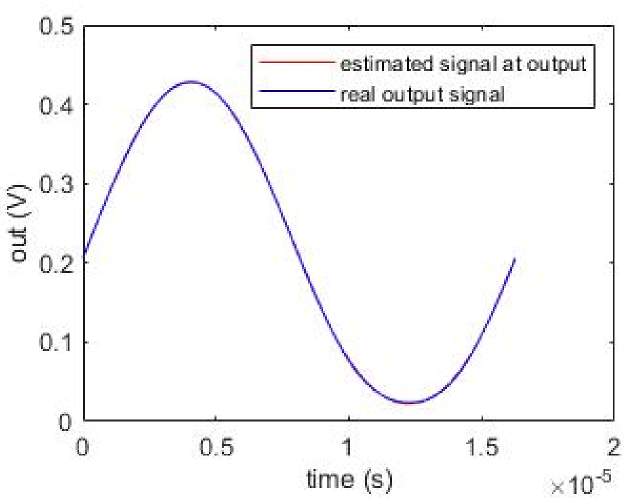

Fig. 4. Volterra series with one input of (1)

Figure 6 represents the Volterra series with two inputs and one output using the small-signal theory applied to the database obtained in this work.

To have a better analysis, the MSE was calculated for each model, and the values are presented in Table 1.

Regarding the coefficients of each model, it is worth noting that the models based on the truncated Taylor series consist of 9 coefficients that were previously extracted. However, these models have the worst accuracy. The Volterra series

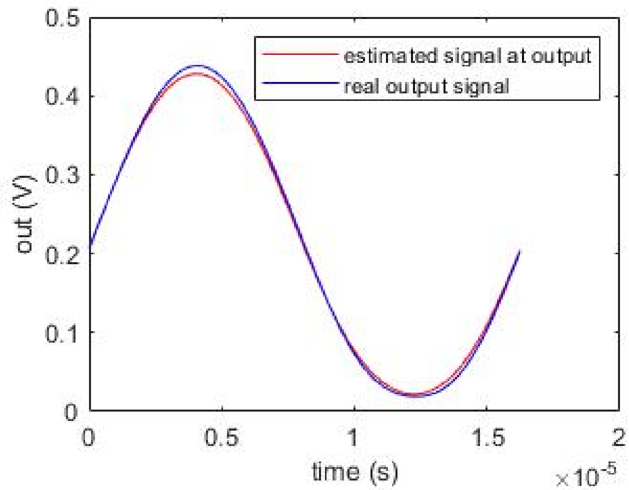

Fig. 5. Modified Volterra Series of (2) applied to the data set with one large-signal input and one small-signal input.

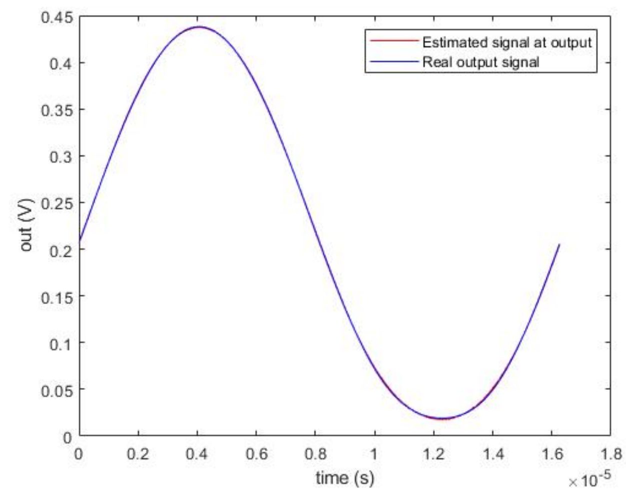

Fig. 6. Modified Volterra Series of (4) applied to the data set with one large-signal input and one small-signal input

TABLE I
MSE OF EACH MODEL

Model	MSE
Volterra series with one input	$7.8526 \cdot 10^{-07}$
Volterra series with two inputs	$8.0823 \cdot 10^{-07}$
Volterra series with with one input as a small signal	$5.4360 \cdot 10^{-07}$
Truncated Taylor series of first order	$3.5188 \cdot 10^{-05}$

with two inputs and one output, using the same parameters as the models used in this work, has 33 coefficients, while the Volterra series with one input as a small signal has 20 coefficients.

V. Conclusion

The Volterra series replaced by a first order Taylor series equation achieves its expected goal of reducing complexity; however, it exhibits low accuracy. This can be observed through its highest MSE among all models.

In comparison to the Volterra series model with two inputs and one output that considers the small-signal theory, it is evident that the former model has lower accuracy.

The model proves to be advantageous in relation to the parameters needed to estimate its output, since it is not necessary even the output of the system with two inputs, which can be useful when it is not possible to have them.

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